

Correlations of flow harmonics in 2.76A TeV Pb–Pb collisions

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Using the event-by-event viscous hydrodynamics VISH2+1 with MC-Glauber, MC-KLN, and AMPT initial conditions, we investigate the correlations of flow harmonics, including the symmetric cumulants $SC^v(m, n)$, the normalized symmetric cumulants $NSC(m, n)$, and the Pearson correlation coefficients $C(v_m^2, v_n^2)$ in 2.76A TeV Pb–Pb collisions. We find $SC^v(m, n)$ is sensitive to both initial conditions and the specific shear viscosity η/s . A comparison with the recent ALICE data show that our hydrodynamic calculations can qualitatively describe the data of $SC^v(3, 2)$ and $SC^v(4, 2)$ for various initial conditions, which demonstrate that v_2, v_4 are correlated and v_2, v_3 are anti-correlated. Meanwhile, the predicted symmetric cumulants $SC^v(5, 2)$, $SC^v(5, 3)$, and $SC^v(4, 3)$ reveal that v_2 and v_5, v_3 and v_5 are correlated, v_3 and v_4 are anti-correlated in most centrality classes. We also find $NSC^v(3, 2)$ and $C(v_3^2, v_2^2)$, which are insensitive to η/s , are mainly determined by corresponding $NSC^e(3, 2)$ and $C(\varepsilon_3^2, \varepsilon_2^2)$ correlators from the initial state. In contrast, other $NSC^v(m, n)$ and $C(v_m^2, v_n^2)$ correlators are influenced by both initial conditions and η/s , which illustrates the non-linear mode couplings in higher flow harmonics with $n \geq 4$.

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I. INTRODUCTION

The main goals of ultrarelativistic heavy-ion collisions at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC) are to produce the strongly interacting Quark-Gluon Plasma (QGP), a deconfined state of quarks and gluons, and to explore its properties [1–4]. The azimuthal anisotropy of produced hadrons is one of the important observables to probe the prosperities of the QGP [5, 6]. It could be characterized by an expansion of the single-particle azimuthal distributions $P(\varphi)$:

$$P(\varphi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} \vec{V}_n e^{-in\varphi} \quad (1)$$

where φ is the azimuthal angle of the emitted particles, \vec{V}_n is the n -th order flow-vector, defined as $\vec{V}_n = v_n e^{in\Psi_n}$. Its magnitude v_n is the n -th order anisotropic flow harmonics and its orientation Ψ_n is the symmetry plane angle. The anisotropic flow harmonics v_n have been studied in great details by many groups (For a recent review, please see [7–9]). The observation of elliptic flow and higher order flow harmonics at RHIC and the LHC, together with the successful descriptions from hydrodynamics and hybrid models, demonstrates that the QGP fireball fluctuates event-by-event and behaves like a nearly perfect liquid with a very small specific shear

viscosity [7–20]. Besides the flow harmonics v_n , additional information for the initial state fluctuations can be obtained by studying the correlations between different order flow-vectors \vec{V}_m and \vec{V}_n . Initially, the study of the correlations between the orientations of different flow-vector was investigated in the observable of v_{2n/Ψ_n} [21–23]. Recently, a full systematic study of so-called “event-plane correlations” was carried by the ATLAS Collaboration [24]. The corresponding hydrodynamic simulations and related theoretical investigations suggest that these new correlations open a new window to probe the details of initial state fluctuations and the transport properties of the QGP [25–29].

In addition, the correlations between different order flow harmonics can be used to further investigate the details of initial-state fluctuations and the hydrodynamic response [30–36]. They also reveal whether different order flow harmonics v_m and v_n are correlated, anti-correlated or uncorrelated. On the experimental side, it is crucial to find an observable that measures the flow harmonics correlations without contributions from the symmetry plane correlations. The first experimental attempt was made by the ATLAS Collaboration in [37]. They investigated the v_m and v_n correlations for events within a given narrow centrality class using an Event-Shape Engineering (ESE), a technique to select events according to the magnitude of reduced flow vector \vec{q}_n [38]. It was observed that, for events within the same centrality class, v_2 is anti-correlated with v_3 and correlated with v_4 [37]. However, this measurement was based on the 2-particle correlations, which might be largely contaminated by non-flow effects. Their method also requires sub-dividing such calculations and modeling resolutions

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associated with ESE due to finite event-wise multiplicities. Considering these constraints, this approach can not be easily utilized in the hydrodynamic simulations with fluctuating initial conditions, as reported in [36].

In Ref. [34], it is suggested to study the correlations between v_m and v_n through the linear correlation function $c(v_m, v_n)$. However, this observable can not be easily accessed in experiments, in which the measurements rely on two- and multi-particle correlations techniques. Later on, a new observable called Symmetric Cumulants $SC^v(m, n)$, which can be measured by the multi-particle cumulant method [30], is proposed to study the correlations between different flow harmonics. It is supposed to be insensitive to the non-flow effects and is free of symmetry plane correlations by design [30]. Recently, $SC^v(4, 2)$ and $SC^v(3, 2)$ are measured by the ALICE Collaboration [33]. Positive values of $SC^v(4, 2)$ and negative values of $SC^v(3, 2)$ are observed at various centrality bins [33], which suggests that v_2 and v_4 are correlated and v_2 and v_3 are anti-correlated. Meanwhile, the HIJING model simulations, which do not include the collective expansion, show that although the non-flow effects lead to non-zero values for both the 2-particle correlations $\langle v_n^2 \rangle$, and the 4-particle correlations $\langle v_m^2 v_n^2 \rangle$, the 4-particle cumulants $SC^v(4, 2)$ and $SC^v(3, 2)$ from HIJING are consistent with zero. This suggests that $SC^v(m, n)$ is an ideal observable that evaluates the correlations between flow harmonics, and is insensitive to non-flow effects.

In this paper, we will investigate the correlations of flow harmonics with event-by-event viscous hydrodynamics VISH2+1 [39, 40]. To study the influences from initial conditions and explore the general properties of the final state correlations, we implement three different initial conditions, namely, MC-Glauber, MC-KLN, and AMPT initial conditions. We will compare our calculated Symmetric Cumulants $SC^v(4, 2)$ and $SC^v(3, 2)$ with the ALICE data and predict other Symmetric Cumulants. We will also study the normalized Symmetric Cumulants $NSC(m, n)$, the Pearson correlation coefficients $C(v_m^2, v_n^2)$ (definitions seen Section II) and explore their sensitivities to initial conditions and the specific shear viscosity.

The paper is organized as follows. Section II introduces the VISH2+1 hydrodynamic model, the setup of calculations and the methodology to calculate the correlations of flow harmonics. Section III presents the results and discussions for the correlations of flow harmonics. Section IV provides a brief summary of this paper.

II. THE MODEL AND THE SETUP OF THE CALCULATIONS

In this paper, we implement event-by-event viscous hydrodynamics VISH2+1 [39, 40] to study the correlations of flow harmonics in 2.76A TeV Pb-Pb collisions. VISH2+1 is a (2+1)-d viscous hydrodynamic code to solve the transport equations of the energy momentum tensor

and the time evolution equations of the shear stress tensor and bulk pressure based on the Israel-Stewart formalism, which simulates the viscous fluid expansion of the hot QCD matter with longitudinal boost-invariance [39]. Around 2011, it was updated to an event-by-event simulation version to further study the initial state fluctuations and final state correlations [41]. The equation of state, initial and decoupling conditions, as well as the transport coefficients are additional inputs of the VISH2+1 code. Generally, VISH2+1 implements the state of the art equation of state EoS-s95p-PCE, which could account for the partially chemical equilibrium effects during the hadronic evolution [42, 43]. Along a decoupling hyper-surface, which is generally defined by a constant temperature T_{dec} , the hydrodynamic information is converted to the final hadron distributions through the Cooper-Fryer formula [44]. Compared with the hybrid model approach that connects viscous hydrodynamics with a hadron cascade model at a switching temperature near T_c (i.e. iEBE-VISHNU [40]), the VISH2+1 simulations implemented in this paper describe both the QGP fluid and the highly dissipative and even off-equilibrium late hadronic stage with fluid-dynamics. With well tuned transport coefficients, decoupling temperature T_{dec} and other related parameters, together with some well-chosen initial conditions (like AMPT [20, 45, 46] and TRENTo [47], etc.), it could fit many related soft hadron data, such as the p_T spectra and different flow harmonics at RHIC and the LHC [41, 43, 45, 48]. Note that this paper does not aim to precisely fit the flow data to extract some information of the hot QCD matter, but focuses on exploring the general properties of the correlations between different flow harmonics. We thus implement the computational efficient VISH2+1 code and leave the more sophisticated but also calculation-time consuming hybrid model simulations to the future study.

To investigate the dependence of flow harmonics correlations on the initial state, we implement three different initial conditions, namely, MC-Glauber, MC-KLN and AMPT initial conditions in the following hydrodynamic calculations. Traditionally, the Glauber model constructs the initial entropy density of the QGP fireball from a mixture of the wounded nucleon and binary collision density profiles [49], and the KLN model assumes the initial entropy density is proportional to the initial gluon density calculated from the corresponding k_T factorization formula [50]. In the Monte-Carlo versions (MC-Glauber and MC-KLN) [51–53], additional initial state fluctuations are introduced through the position fluctuations of individual nucleons inside the colliding nuclei. For the AMPT initial conditions [20, 45, 46], the fluctuating energy density profiles are constructed from the energy decompositions of individual partons, which fluctuate in both momentum and position space. Compared with the MC-Glauber and MC-KLN initial conditions, the additional Gaussian smearing parameter in the AMPT initial conditions makes the typical initial fluctuation scales changeable, which helps to achieve better hydrodynamic descriptions of the

anisotropic flow data.

Considering the conversion from initial entropy to final multiplicity of all charged hadrons, the centrality is determined via the distribution of total entropies of the fluctuating initial profiles. In order to explore the shear viscosity dependence of flow harmonic correlations, we choose two values of η/s , 0.08 and 0.20, for **MC-Glauber** and **MC-KLN** initial conditions, and 0.08 and 0.16 for **AMPT** initial conditions¹. For each initial condition and η/s , the normalization factors of the initial entropy density profiles and the hydrodynamic starting time τ_0 are respectively tuned to fit the multiplicity and p_T spectra of all charged hadrons in the most central Pb-Pb collisions [20]. Following [20, 48], the decoupling temperature T_{dec} is set to 120 MeV, which could roughly describe the slope of the p_T spectra of protons in the most central collisions. To simplify the theoretical investigations, we set the bulk viscosity, net baryon density and the heat conductivity to zero in the following calculations.

After the hydrodynamic evolution and thermal freeze-out, the anisotropic flow coefficients v_n and its corresponding symmetry plane angle Ψ_n can be calculated as [12, 40]:

$$v_n e^{in\Psi_n} = \frac{\int p_T dp_T d\varphi e^{in\varphi} \frac{dN_{ch}^3}{p_T dp_T d\varphi d\eta}}{\int p_T dp_T d\varphi \frac{dN_{ch}^3}{p_T dp_T d\varphi d\eta}} \quad (2)$$

where φ is the azimuthal angle of the emitted particles, n is the order of the flow harmonics.

With v_n obtained from the above equation, one can calculate the Symmetric Cumulants, $SC^v(m, n)$ defined as the following [33]:

$$SC^v(m, n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle. \quad (3)$$

To further evaluate the correlations of flow harmonics, one could define the normalized Symmetric Cumulants:

$$NSC^v(m, n) = \frac{SC^v(m, n)}{\langle v_m^2 \rangle \langle v_n^2 \rangle} = \frac{\langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle}{\langle v_m^2 \rangle \langle v_n^2 \rangle} \quad (4)$$

Compared with $SC^v(m, n)$, $NSC^v(m, n)$ reflects the relative correlation between v_m and v_n , which is expected to be insensitive to the magnitudes of v_m and v_n .

Alternatively, the correlations between flow harmonics v_m and v_n can be investigated via the Pearson correlation coefficient, which has been widely used to evaluate the degree of linear dependence between two variables [34,

56]. The Pearson correlation coefficient is defined as:

$$C(v_m^2, v_n^2) = \rho_{v_m^2, v_n^2} = \frac{\langle (v_m^2 - \langle v_m^2 \rangle)(v_n^2 - \langle v_n^2 \rangle) \rangle}{\sigma_{v_m^2} \sigma_{v_n^2}} \\ = \frac{\langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle}{\sqrt{\langle v_m^4 \rangle - \langle v_m^2 \rangle^2} \sqrt{\langle v_n^4 \rangle - \langle v_n^2 \rangle^2}}. \quad (5)$$

where σ_{v_m} stands for the standard deviation of v_m distributions (In the flow language, it is also the flow fluctuations of v_m). Generally speaking, $C(v_m^2, v_n^2) = 1$ or -1 means that the variables v_m and v_n are total linearly correlated or anti-correlated, $C(v_m^2, v_n^2) = 0$ means v_m and v_n are uncorrelated.

Correspondingly, one could also investigate the correlations between different eccentricity coefficients. For a fluctuating initial profile, the eccentricity coefficients ε_n and the initial symmetry plane (participant plane) angle Φ_n are defined as [12, 40]:

$$\varepsilon_n e^{in\Phi_n} = -\frac{\int r dr d\varphi r^n e^{in\varphi} e(r, \varphi)}{\int r dr d\varphi r^n e(r, \varphi)}, \quad (6)$$

where $e(r, \varphi)$ is the initial energy density in the transverse plane, φ is azimuthal angle and n is the order of the coefficient. Analogous to Eqs. (3-5), we propose the corresponding correlators $SC^\varepsilon(m, n)$, $NSC^\varepsilon(m, n)$, and $C(\varepsilon_m^2, \varepsilon_n^2)$ for the initial state, which are defined as following:

$$SC^\varepsilon(m, n) = \langle \varepsilon_m^2 \varepsilon_n^2 \rangle - \langle \varepsilon_m^2 \rangle \langle \varepsilon_n^2 \rangle, \quad (7)$$

$$NSC^\varepsilon(m, n) = \frac{SC^\varepsilon(m, n)}{\langle \varepsilon_m^2 \rangle \langle \varepsilon_n^2 \rangle} = \frac{\langle \varepsilon_m^2 \varepsilon_n^2 \rangle - \langle \varepsilon_m^2 \rangle \langle \varepsilon_n^2 \rangle}{\langle \varepsilon_m^2 \rangle \langle \varepsilon_n^2 \rangle}, \quad (8)$$

and

$$C(\varepsilon_m^2, \varepsilon_n^2) = \rho_{\varepsilon_m^2, \varepsilon_n^2} = \frac{\langle (\varepsilon_m^2 - \langle \varepsilon_m^2 \rangle)(\varepsilon_n^2 - \langle \varepsilon_n^2 \rangle) \rangle}{\sigma_{\varepsilon_m^2} \sigma_{\varepsilon_n^2}} \\ = \frac{\langle \varepsilon_m^2 \varepsilon_n^2 \rangle - \langle \varepsilon_m^2 \rangle \langle \varepsilon_n^2 \rangle}{\sqrt{\langle \varepsilon_m^4 \rangle - \langle \varepsilon_m^2 \rangle^2} \sqrt{\langle \varepsilon_n^4 \rangle - \langle \varepsilon_n^2 \rangle^2}} \quad (9)$$

III. RESULTS AND DISCUSSION

Before investigating the correlations between different flow harmonics, we firstly calculate the p_T -integrated flow v_2 , v_3 , and v_4 of all charged hadrons in 2.76A TeV Pb-Pb collisions, using event-by-event viscous hydrodynamics **VISH2+1** with different combinations of initial conditions and specific shear viscosity. The comparison with the ALICE data [10] are shown in Fig. 1. It demonstrates that, for the viscous hydrodynamic simulations with a uniform η/s , neither **MC-Glauber** nor **MC-KLN** initial conditions can simultaneously describe v_2 , v_3 , and v_4 , as once reported in [41]. More specifically, for **MC-Glauber** initial conditions, **VISH2+1** with $\eta/s = 0.08$ could nicely

¹ We have noticed, in peripheral Pb-Pb collisions, **VISH2+1** simulations with **AMPT** initial conditions and $\eta/s = 0.20$ are roughly out of the validity regime of hydrodynamics due to large viscous corrections [40, 54, 55]. We thus choose a smaller value of the specific shear viscosity $\eta/s = 0.16$ for the corresponding comparison runs.

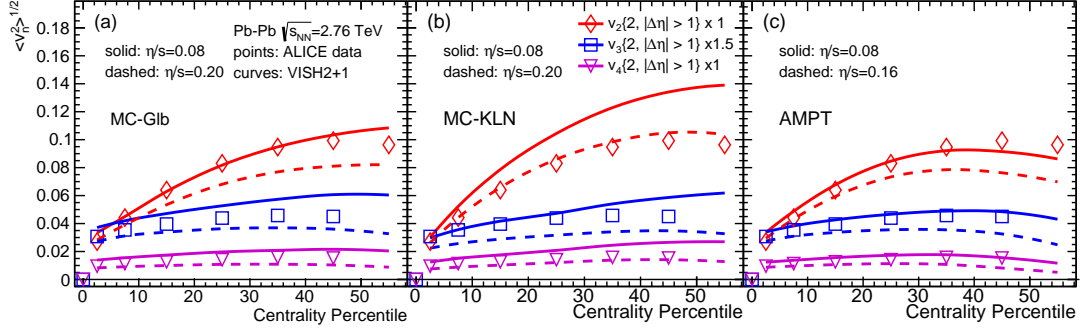


FIG. 1. (Color online) Integrated flow v_2 (red curves, $\times 1$), v_3 (blue curves, $\times 1.5$), and v_4 (magenta curves, $\times 1$) of all charged hadrons in 2.76 A TeV Pb-Pb collisions, calculated from VISH2+1 with MC-Glauber (left), MC-KLN (middle), and AMPT (right) initial conditions, together with a comparison to the ALICE data [10].

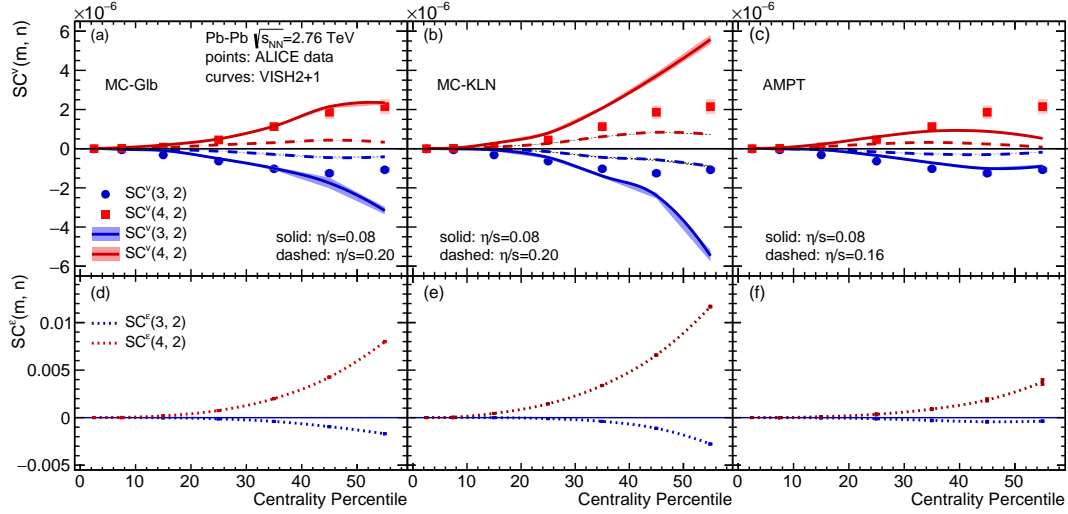


FIG. 2. (Color online) Top panels: Symmetric cumulants $SC^v(3, 2)$ and $SC^v(4, 2)$ in 2.76 A TeV Pb-Pb collisions, calculated from VISH2+1 with MC-Glauber (a), MC-KLN (b), and AMPT (c) initial conditions and with different η/s . The measured $SC^v(3, 2)$ and $SC^v(4, 2)$ from the ALICE Collaboration are also presented here, which are taken from [33]. Bottom panels: Symmetric cumulants of the initial eccentricity coefficients $SC^e(3, 2)$ and $SC^e(4, 2)$ for these three initial conditions.

fit the integrated v_2 from central to semi-peripheral collisions, but overestimates v_3 and v_4 for the same centrality classes. For MC-KLN initial conditions, VISH2+1 with $\eta/s = 0.20$ reproduces the integrated flow v_2 but underestimates v_3 and v_4 for the presented centrality classes. Compared with these two results, hydrodynamic calculations with AMPT initial conditions improves the descriptions of v_n ($n = 2, 3, 4$) with an additional smearing factor σ during the initial energy depositions [20]. Panel (c) shows that VISH2+1 with AMPT initial condition and $\eta/s = 0.08$ roughly describe v_2 , v_3 and v_4 from central to semi-peripheral collisions². Note that, for AMPT initial

conditions, we do not finely tune η/s and other related parameters to obtain the best fit of v_n ($n=2, 3, 4$), but continue to use one of the “standard” specific shear viscosity $\eta/s = 0.08$ as used for MC-Glauber and MC-KLN initial conditions.

Using the same inputs and parameter sets, we calculate the symmetric cumulants $SC^v(m, n)$ in 2.76A TeV Pb-Pb collisions with VISH2+1. The upper panels of Fig. 2 show the comparisons between our model calculations and the ALICE measurements. For all three initial conditions and different values of η/s , VISH2+1 could re-

² Ref. [45] showed better descriptions of v_n ($n = 2, 3, 4$) for the VISH2+1 simulations with AMPT initial conditions, especially for the centrality-dependent v_2 . Compared with our calculations, which define the centrality bins through the distributions of ini-

tial total entropies, their centrality bins are cut by the empirical formula of AMPT $c = \pi b^2/\sigma$ [57, 58]. Since this paper is not aim to study the properties of flow harmonics, quantitatively, we continue to use the early parameters sets of AMPT as used in [45, 57, 58], rather than fine-tune them to obtain a better description of the centrality dependent v_2 .

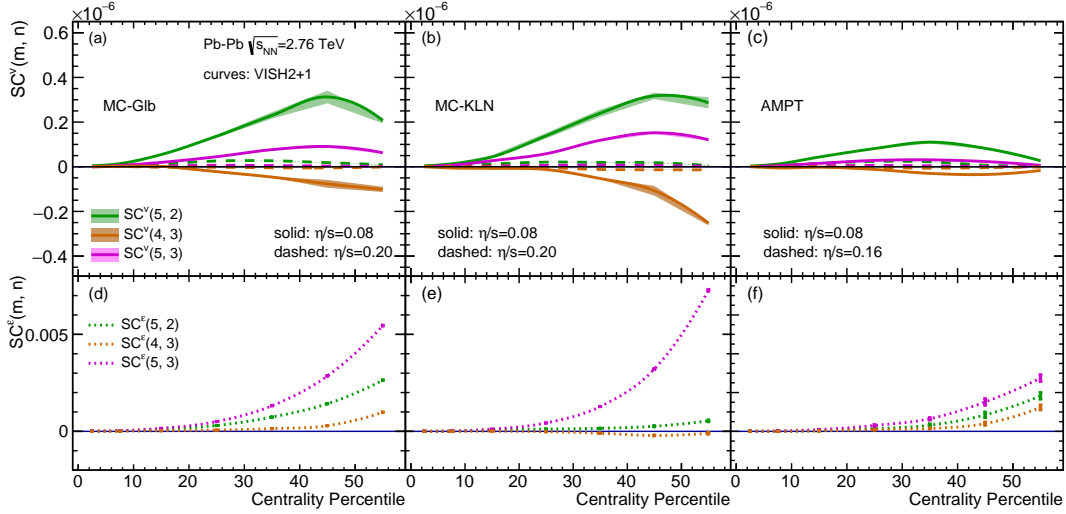


FIG. 3. (Color online) Similar to Fig. 2, but for the predicted symmetric cumulants $SC^v(5, 2)$, $SC^v(5, 3)$ and $SC^v(4, 3)$ in 2.76 A TeV Pb–Pb collisions, together with the calculated symmetric cumulants of the initial eccentricity coefficients $SC^e(5, 2)$, $SC^e(5, 3)$ and $SC^e(4, 3)$ for MC–Glauber, MC–KLN, and AMPT initial conditions, respectively.

produce the typical features of the correlations between different flow harmonics, which shows negative values of $SC^v(3, 2)$ and positive values of $SC^v(4, 2)$. Therefore, both the experimental data and hydrodynamic calculations suggest that v_2 and v_3 are anti-correlated, while v_2 and v_4 are correlated. It also indicates that, for a specific event with larger value of v_2 above the event sample averaged $\langle v_2 \rangle$, the probability of finding smaller value of v_3 below $\langle v_3 \rangle$ and the probability of finding larger value of v_4 above $\langle v_4 \rangle$ are both enhanced. Fig. 2 also demonstrates that $SC^v(3, 2)$ and $SC^v(4, 2)$ are sensitive to the specific shear viscosity η/s of the expanding fireball. For these three initial conditions, the anti-correlations between v_2 and v_3 and the correlations between v_2 and v_4 are both suppressed by the larger value of η/s . This is due to the fact that the correlation strength of $SC^v(m, n)$ depends on the magnitudes of v_m and v_n . Larger η/s leads to a larger suppression of the flow harmonics v_n , which results in smaller values of $SC^v(m, n)$. This agrees with the conclusion from the transport model calculations [30], which shows that stronger (anti-)correlations of $SC^v(m, n)$ are produced when using larger partonic cross sections (corresponding to smaller η/s [59]).

In Fig. 2 (d), (e) and (f), we plot the symmetric cumulants of the initial eccentricity coefficients $SC^e(3, 2)$ and $SC^e(4, 2)$. For each initial condition, $SC^e(3, 2)$ shows negative values and $SC^e(4, 2)$ shows positive values, which demonstrates that ε_2 and ε_3 are anti-correlated, and ε_2 and ε_4 are correlated. The upper and lower panels of Fig. 2 also reveal that, although the signs of $SC^v(3, 2)$ and $SC^v(4, 2)$ are the same as the signs of $SC^e(3, 2)$ and $SC^e(4, 2)$, respectively, the correlation strength of them are strongly influenced by the viscous corrections of the QGP fireball. In peripheral collisions, both $SC^e(3, 2)$ and $SC^e(4, 2)$ shows larger correlation strength for these

three initial conditions. However, the viscous fluid expansion has limited power to effectively convert the initial state correlations into the final state ones due to the largely reduced evolution time.

Although none of the above combination of initial conditions and η/s can quantitatively describes the data from ALICE, it is still impressive that event-by-event hydrodynamic simulations can correctly capture the sign of $SC^v(3, 2)$, $SC^v(4, 2)$ and roughly describe the centrality dependence. Ref. [29] also showed that, although the EKRT+viscous hydrodynamic calculations with a temperature dependent $\eta/s(T)$ can nicely describe the centrality dependent integrated flow v_2 , v_3 and v_4 , the related model calculations can not quantitatively reproduce the centrality dependent $SC^v(3, 2)$ and $SC^v(4, 2)$ measurements. Meanwhile, the calculation from HIJING simulations shows almost zero values of $SC^v(3, 2)$ and $SC^v(4, 2)$ [33]. These different theoretical calculations suggest that the correlations between different flow harmonics, i.e. $SC^v(3, 2)$ and $SC^v(4, 2)$, reflect the hydrodynamics response of the initial state correlations, which are more sensitive to the details of theoretical models than the individual v_n coefficients alone.

In Fig. 3, we predict the centrality dependent $SC^v(m, n)$ for other combinations of flow harmonics ($(m, n) = (5, 2)$, $(5, 3)$ and $(4, 3)$), together with the calculations of $SC^e(m, n)$ for the corresponding initial eccentricity coefficient pairs. For all three initial conditions, $SC^v(5, 2)$ and $SC^v(5, 3)$ are positive, and $SC^v(4, 3)$ are negative, which reveals that v_2 and v_5 , v_3 and v_5 are correlated, while v_3 and v_4 are anti-correlated. Similar to $SC^v(3, 2)$ and $SC^v(4, 2)$, the magnitudes of $SC^v(5, 2)$, $SC^v(5, 3)$ and $SC^v(4, 3)$ are sensitive to the specific shear viscosity of QGP. Their correlation strengths become weaker with the increase of η/s . We also observe that

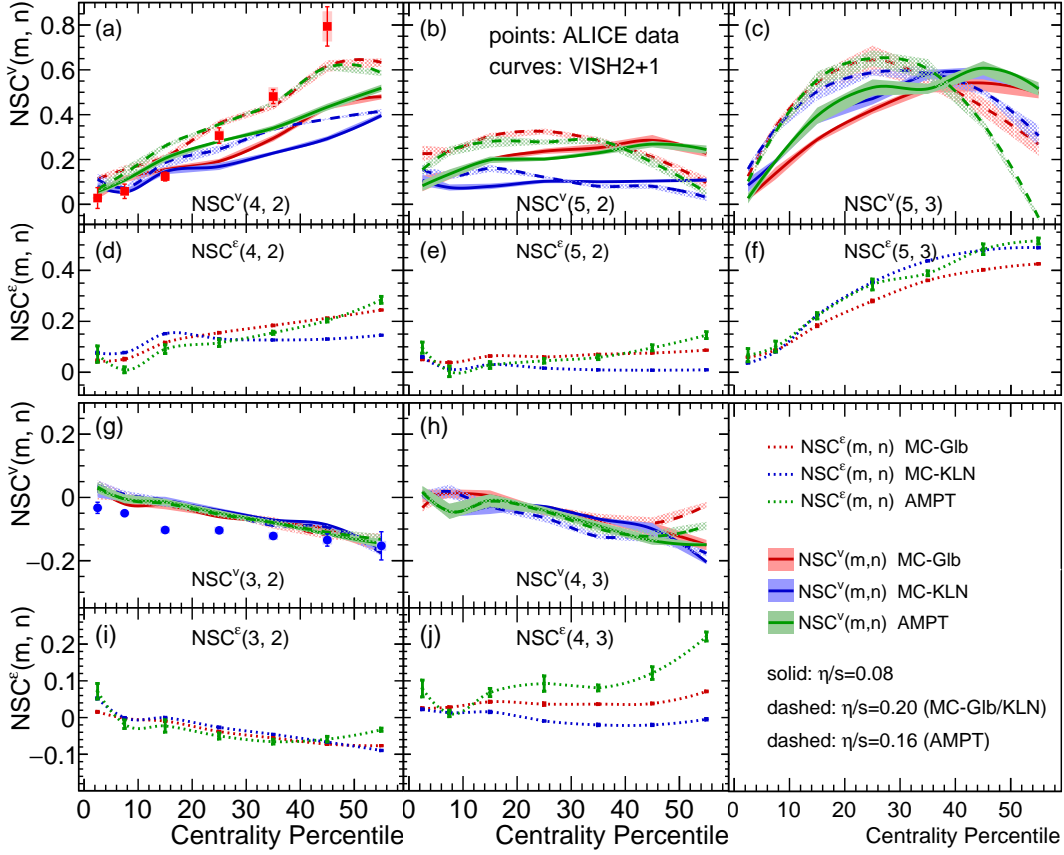


FIG. 4. (Color online) Normalized symmetric cumulants $NSC^v(m, n)$ and normalized symmetric cumulants of the initial eccentricity coefficients $NSC^e(m, n)$ in 2.76A TeV Pb-Pb collisions.

the signs of $SC^v(5, 2)$ and $SC^v(5, 3)$ are the same as the corresponding initial state correlators $SC^e(5, 2)$ and $SC^e(5, 3)$. While, $SC^v(4, 3)$ and $SC^e(4, 3)$ present opposite signs for MC-Glauber and AMPT initial conditions. In Refs. [60–62], it has been found that the v_4 signals are influenced by both ε_4 and ε_2^2 of the initial conditions, where the ε_2^2 term makes the dominant contributions in non-central collisions [63]. As a result, the anti-correlation between ε_2 and ε_3 significantly contributes to $SC^v(4, 3)$, leading to a changing sign of $SC^v(4, 3)$, when compared with $SC^e(4, 3)$ for MC-Glauber and AMPT initial conditions.

In order to further study the correlations between flow harmonics, we normalize $SC^v(m, n)$ and $SC^e(m, n)$ with $\langle v_m^2 \rangle \langle v_n^2 \rangle$ and $\langle \varepsilon_m^2 \rangle \langle \varepsilon_n^2 \rangle$ according to Eqs. (4) and (8), respectively. The normalized symmetric cumulants $NSC^v(m, n)$ and the corresponding initial state normalized correlators $NSC^e(m, n)$ are plotted in Fig. 4, where panel (a) and (g) also show the corresponding measurements from ALICE [33]. We find $NSC^v(4, 2)$, $NSC^v(5, 2)$, and $NSC^v(5, 3)$ are sensitive to both initial conditions and the specific shear viscosity η/s . More specifically, they show sizable changes to the change of η/s for certain initial condition. Meanwhile, they are also influenced by the initial conditions. Their corresponding

NSC^e correlators also separate in different initial conditions. For AMPT and MC-Glauber initial conditions, the VISH2+1 calculations roughly fit $NSC^v(4, 2)$ data with $\eta/s = 0.16$ and $\eta/s = 0.2$, which demonstrates that such normalized symmetric cumulants can help to constrain the QGP viscosity. We also find, for these three investigated initial conditions, $NSC^e(3, 2)$ are almost overlap from central to semi-central collisions which only slightly split in peripheral collisions. At the same time, the normalized symmetric cumulants $NSC^v(3, 2)$ is insensitive to the QGP shear viscosity since v_2 and v_3 are roughly proportional to ε_2 and ε_3 . As a result, these different $NSC^v(3, 2)$ curves in Fig. 4 (g) are almost overlap with each other, which also roughly fit the normalized ALICE data. Similar to the case of $SC^v(4, 3)$ and $SC^e(4, 3)$ in Fig. 3, $NSC^v(4, 3)$ does not follow the sign of $NSC^e(4, 3)$ for both MC-Glauber and AMPT initial conditions due to the nonlinear and dominant contribution of ε_2^2 to v_4 from semi-central to peripheral collisions. Panel (h) and (j) also show that, although $NSC^e(4, 3)$ is strongly depends on the initial conditions, $NSC^v(4, 3)$ is not very sensitive to the initial conditions, which roughly overlaps from central to semi-peripheral collisions for different η/s . In a recent work, the $NSC^v(m, n)$ are expressed in terms of event-plane correlations and moments of v_2 and v_3

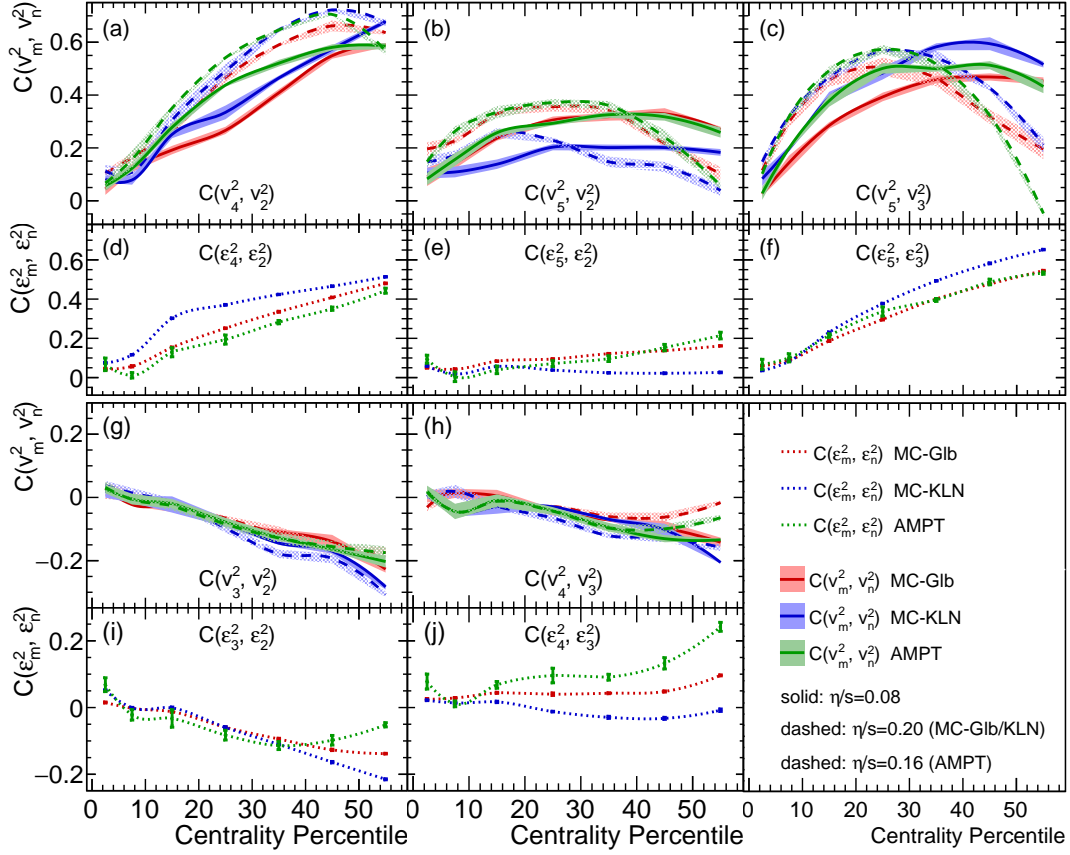


FIG. 5. (Color online) The Pearson correlation coefficients of flow harmonics $C(v_m^2, v_n^2)$ and the Pearson correlation coefficients of the initial eccentricity coefficients $C(\epsilon_m^2, \epsilon_n^2)$ in 2.76A TeV Pb-Pb collisions.

(see Eqs. (8) and (9) in [35]). Considering the relative flow fluctuations of v_3 is stronger than v_2 , one expects that $\langle v_2^4 \rangle / \langle v_2^2 \rangle^2$ is smaller than $\langle v_3^4 \rangle / \langle v_3^2 \rangle^2$ [31], which in the end gives smaller values for $NSC^v(5, 2)$ than $NSC^v(5, 3)$. This is indeed observed in Fig. 4 (b) and (c). On the other hand, it was predicted the $NSC^v(m, n)$ correlators that involve v_4 or v_5 increase with η/s in the same way as the event-plane correlations [26, 61]. This seems agree with what shown in panel (a) as well as the cases from central to semi-peripheral collisions in panel (b) and (c) of Fig. 4, but in contrast to the results for peripheral collisions in panel (b) and (c). Examinations on the Eqs. (8) and (9) in [35] and the corresponding assumption used in nonlinear hydrodynamic response phenomena with future experimental data are necessary to explain the difference between our results and those theoretical predictions in [35].

Besides $NSC^v(m, n)$, one can also investigate the correlations between different flow harmonics through the Pearson correlation coefficients $C(v_m^2, v_n^2)$ defined by Eq. (5). As introduced in Sec. II, $C(v_m^2, v_n^2)$ could further evaluate the linear relationship between v_m and v_n . In Fig. 5, we plot the centrality dependent $C(v_m^2, v_n^2)$ calculated from the VISH2+1 with different initial conditions and η/s , together with the corresponding initial

state correlators $C(\epsilon_m^2, \epsilon_n^2)$. We find that the absolute values of all $C(v_m^2, v_n^2)$ and $C(\epsilon_m^2, \epsilon_n^2)$ do not equal to 1, which indicate none of the (v_m^2, v_n^2) , $(\epsilon_m^2, \epsilon_n^2)$ pairs are linearly correlated or anti-correlated. For different (m, n) pairs, $C(v_m^2, v_n^2)$ shows similar dependence on initial conditions and the specific shear viscosity η/s as the case of $NSC^v(m, n)$. More specifically, $C(v_4^2, v_2^2)$, $C(v_5^2, v_2^2)$, and $C(v_5^2, v_3^2)$ strongly depend on both initial conditions and η/s , however, $C(v_3^2, v_2^2)$ is insensitive to initial conditions and η/s . Meanwhile, the corresponding $C(\epsilon_3^2, \epsilon_2^2)$ from different initial conditions almost overlap with each other from central to semi-peripheral collisions. Although $C(\epsilon_4^2, \epsilon_3^2)$ from different initial conditions show significant separations, $C(v_4^2, v_3^2)$ from our model calculations is not very sensitive to initial conditions since v_4 is largely influence by ϵ_2^2 from semi-central to peripheral collisions.

Both Fig. 4 and Fig. 5 demonstrate that, from central to semi-peripheral collisions, the normalized symmetric cumulant $NSC^v(3, 2)$ and Pearson correlation coefficient $C(v_3^2, v_2^2)$ are insensitive to the specific shear viscosity η/s for various initial conditions. In Fig. 6, we directly compare $NSC^v(3, 2)$ and $C(v_3^2, v_2^2)$ with the corresponding initial state correlators $NSC^\epsilon(3, 2)$ and $C(\epsilon_3^2, \epsilon_2^2)$, respectively. We observe for each initial condi-

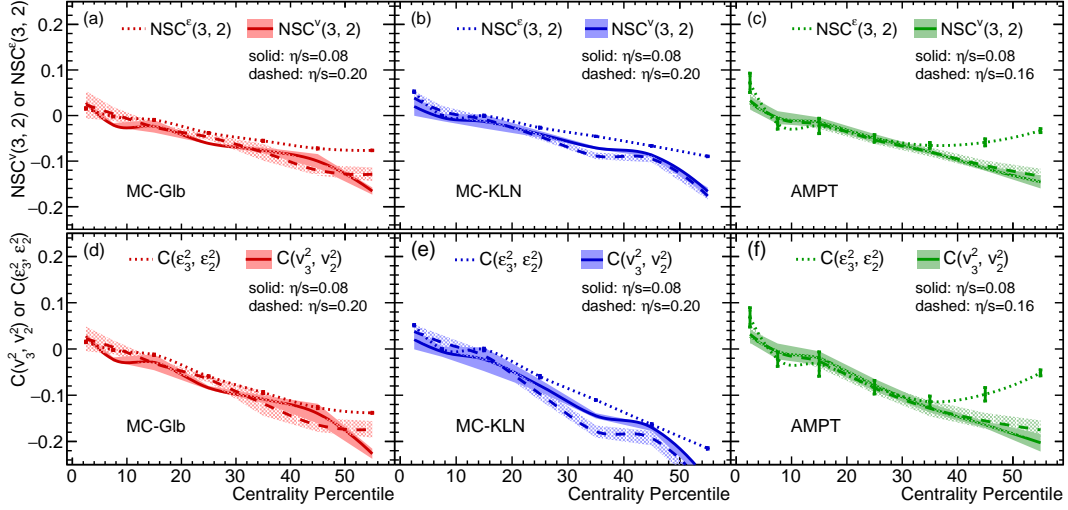


FIG. 6. (Color online) Up panels: a comparison of the normalized symmetric cumulants $NSC^v(3, 2)$ and the normalized symmetric cumulants of the initial eccentricity coefficients $NSC^e(3, 2)$ for MC-Glauber (left), MC-KLN (middle), and AMPT (right) initial conditions. Lower panels: a similar comparison of the corresponding Pearson correlation coefficients $C(v_3^2, v_2^2)$ and $C(\epsilon_3^2, \epsilon_2^2)$.

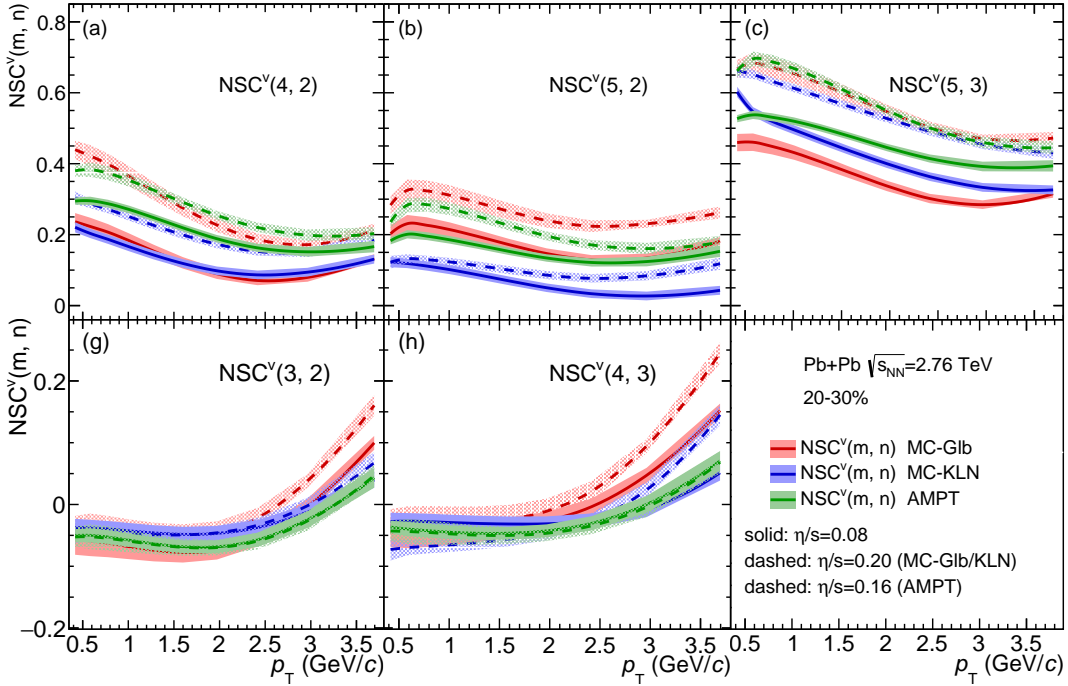


FIG. 7. (Color online) p_T dependent normalized symmetric cumulants $NSC^v(m, n)$ at 20-30% centrality in 2.76 A TeV Pb-Pb collisions, calculated from VISH2+1 with MC-Glauber (red), MC-KLN (blue), and AMPT (green) initial conditions.

tion, $NSC^v(3, 2)$ are almost overlap with the $NSC^e(3, 2)$ within the statistical uncertainties from central to semi-central collisions, despite of the input η/s . Similarly, $C(v_3^2, v_2^2)$ and $C(\epsilon_3^2, \epsilon_2^2)$ also roughly overlap from central to semi-central collisions. These results demonstrate that the $NSC^v(3, 2)$ and $C(v_3^2, v_2^2)$ are mainly determined by corresponding correlators $NSC^e(3, 2)$ and $C(\epsilon_3^2, \epsilon_2^2)$ from

the initial state.

Figure 7 presents $NSC^v(m, n)$ as a function of p_T in 20-30% Pb-Pb collisions. Besides different sensitivities to initial conditions and η/s as discussed above, we notice that $NSC^v(m, n)$ also shows different p_T dependence. More specifically, $NSC^v(4, 2)$, $NSC^v(5, 2)$ and $NSC^v(5, 3)$ are positive for the entire p_T range, while

$NSC^v(3,2)$ and $NSC^v(4,3)$ are negative at low p_T but change to positive for $p_T > 3$ GeV/ c . The trend is qualitatively agreed with the conclusion obtained with linear correlation function $c(v_m, v_n)$ reported in [34], although different moments of v_m and v_n are used. Future investigations on non-linear hydrodynamic response in higher flow harmonics will help us to better understand the observed different behaviors of p_T dependent $NSC^v(m,n)$.

IV. SUMMARY

In this paper, we investigate the correlations of flow harmonics in 2.76A TeV Pb-Pb collisions using the event-by-event viscous hydrodynamics VISH2+1 with MC-Glauber, MC-KLN, and AMPT initial conditions. We found the symmetric cumulants $SC^v(m,n)$ are sensitive to both initial conditions and the specific shear viscosity η/s . When compared to the ALICE data, our VISH2+1 calculations could qualitatively describe $SC^v(3,2)$ and $SC^v(4,2)$ for different initial conditions, which demonstrate that v_2 and v_4 are correlated and v_2 and v_3 are anti-correlated. We also predicted other symmetric cumulants with different (m,n) combinations and found v_2 and v_5 , v_3 and v_5 are correlated, v_3 and v_4 are anti-correlated at various centralities.

In addition, we investigate the normalized symmetric cumulants $NSC^v(m,n)$ and the Pearson correlation coefficients $C(v_m^2, v_n^2)$. We found $NSC^v(3,2)$ and $C(v_3^2, v_2^2)$ are mainly determined by corresponding $NSC^\epsilon(3,2)$ and $C(\epsilon_3^2, \epsilon_2^2)$ correlators from the initial state, which roughly overlap from central to semi-peripheral collisions for the three initial conditions used in our calculations. Furthermore, $NSC^v(3,2)$ and $C(v_3^2, v_2^2)$ are insensitive to the specific shear viscosity η/s in the hydrodynamic simulations, since both v_2 and v_3 are approximately linearly response to ϵ_2 and ϵ_3 of the initial state. In contrast, $NSC^v(4,2)$, $NSC^v(5,2)$, $NSC^v(5,3)$, as well as $C(v_4^2, v_2^2)$, $C(v_5^2, v_2^2)$, and $C(v_5^2, v_3^2)$, are sensitive to both initial conditions and η/s . We also found, for both MC-Glauber and AMPT initial conditions, $NSC^v(4,3)$ does

not follow the sign of $NSC^\epsilon(4,3)$ due to the nonlinear mode couplings between v_4 and v_2 . Correspondingly, although $NSC^\epsilon(4,3)$ is strongly depends on the initial conditions, $NSC^v(4,3)$ curves are not very sensitive to the initial conditions, which are also roughly overlap for different initial conditions and specific shear viscosity η/s from central to semi-peripheral collisions. We also studied the p_T dependent normalized symmetric cumulants and observed that $NSC^v(4,2)$, $NSC^v(5,2)$ and $NSC^v(5,3)$ are positive while $NSC^v(3,2)$ and $NSC^v(4,3)$ change sign from negative to positive at $p_T \sim 3$ GeV/ c .

Compared to individual v_n coefficients, the correlations between different flow harmonics are more sensitive to the details of theoretical calculations. Future experimental measurements on the predicted observables, including symmetric cumulants $SC^v(5,2)$, $SC^v(5,3)$, $SC^v(4,3)$ and the normalized symmetric cumulants $NSC(m,n)$, the Pearson correlation coefficients $C(v_m^2, v_n^2)$, and the further related hydrodynamic investigations will shed new light into the nature of the initial state fluctuations and the properties of the QGP fireball created in the ultra-relativistic heavy ion collisions.

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